c 2017

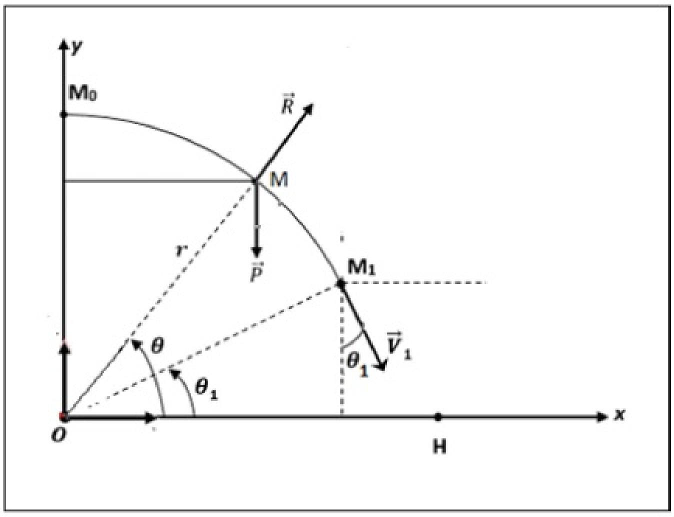
Détails

Catégorie : Mécanique

3.1.

3.1.1. Bilan des forces :

$\vec{P}\,(poids)$ et Invalid Equation .



3.1.2. TCI : $\vec{P}+\vec{R}=m\vec{a}$

Projetons suivant la normale :

$P_N + R_N = ma_n \Rightarrow$

$m.g.sin\theta - R = ma_n$ or

$an = \dfrac{V^2}{r} \Rightarrow$

$$R = m(g.sin\theta - \frac{V^2}{r}$$

3.1.3.

$$T.E.C \Rightarrow E_{c(M)} - E_{c(M_0)} = W_{\vec{P}} + W_{\vec{R}}$$

$$\Rightarrow \frac{1}{2}mV^2 = mgh \quad avec \quad h = r(1-sin\theta)$$

$$\Rightarrow \frac{1}{2}mV^2 = mgr(1-sin\theta)$$

$$\Rightarrow V^2 = 2.g.r(1-sin\theta)$$

3.1.4. Lorsque le mobile quitte la piste en

$$M_1\ : \theta=\theta_1\,;\, V\ = V_1\, et\, R\ =\ 0 \Rightarrow m\left(g.sin\theta - \frac{V^2}{r}\right) = 0 \Rightarrow$$

$$\Rightarrow g.sin\theta_1 - \frac{V_1^2}{r} = 0 \Rightarrow V_1^2\ =\ g.r.sin\theta_1\ =\ 2.g.r(1-sin\theta_1) \Rightarrow sin\theta_1\ = \frac{2}{3} \Rightarrow \theta_1\ =\ 41,8^\circ$$

Expression de $V_1\ :\ V_1^2 = g.r.sin\theta_1\ =\ g.r.\frac{2}{3} \Rightarrow V_1 = \sqrt{\frac{2}{3}g.r}$

3.2.

3.2.1. Expression des composantes de $\vec{V_1}$

$$\vec{V_1}\begin{cases} V_{1x} &= V_1.sin\theta_1 \\ V_{1y} &= -V_1.cos\theta_1 \end{cases}$$

3.2.2. Equations horaires : TCI :

$$\vec{P} = m\vec{a} \Rightarrow m\vec{a} = m\vec{g} \Rightarrow \vec{a} = \vec{g} \Rightarrow \vec{a}\begin{cases} a_x &= 0 \\ a_y &= -g \end{cases} \quad \vec{V}\begin{cases} V_x &= V_1.sin\theta_1 \\ V_y &= -gt - V_1.cos\theta_1 \end{cases}$$

$$\Rightarrow \overrightarrow{OM} \left\{ \begin{array}{lll} x & = & V_1.sin\theta_1.t + r.cos\theta_1 \\ & & \\ y & = & -\frac{1}{2}.g.t^2 - V_1.cos\theta_1.t + r.sin\theta_1 \end{array} \right.$$

Equation de la trajectoire est :

$$y = -\frac{g}{2(V_1.sin\theta_1)}.(x - r.cos\theta_1)^2 - \frac{x - r.cos\theta_1}{tan\theta_1} + r.sin\theta_1.$$

3.2.3. Expression de OH : au point H on a y = 0

$$-\frac{g}{2(V_1.sin\theta_1)^2}.(x - r.cos\theta_1)^2 - \frac{(x - r.cos\theta_1)}{tan\theta_1} + r.sin\theta_1 = 0$$

Posons $u = (x - r.cos\theta_1) \Rightarrow -\frac{g}{2(V_1.sin\theta_1)^2}.u^2 - \frac{u}{tan\theta_1} + r.sin\theta_1 = 0$

$$\Delta = \frac{1}{(tan\theta_1)^2} + \frac{4gr}{2V_1^2.sin\theta_1} = \frac{1}{(tan\theta_1)^2} + \frac{2gr}{\frac{1}{2}g.r.sin\theta_1} = \frac{1}{(tan\theta_1)^2} + \frac{3}{sin\theta_1} = \frac{1}{(tan\theta_1)^2} + \frac{9}{2} \Rightarrow$$

$$\left\{ \begin{array}{lllll} u_1 & = & \frac{\frac{1}{tan\theta_1} - \sqrt{\Delta}}{-\frac{g}{(V_1.sin\theta_1)^2}} & > & 0 \\ & & & & \\ u_2 & = & \frac{\frac{1}{tan\theta_1} + \sqrt{\Delta}}{-\frac{g}{(V_1.sin\theta_1)^2}} & < & 0 \end{array} \right\} .u_1 = \frac{(V_1.sin\theta_1)^2 \left(\sqrt{\Delta} - \frac{1}{tan\theta_1} \right)}{g} = \frac{\frac{2}{3}gr.\frac{4}{9} \left(\sqrt{\Delta} - \frac{1}{tan\theta_1} \right)}{g}$$

$$u_1 = \frac{8}{27}r \left(\sqrt{\Delta} - \frac{1}{tan\theta_1} \right) = 0,379.r$$

or $u = (x - r.cos\theta_1) \Rightarrow x = u + r.cos\theta_1 = 0,379.r + r.cos41,8^\circ \Rightarrow x = 1,12.r$

Expression de la distance OH en fonction de r : OH = 1,12.r